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TRANSVERSE POLARIZATION OF BETA PARTICLES
IN FIRST FORBIDDEN BETA DECAY

A THESIS

Presented to
the Faculty of the Graduate Division


By
Robert Edgar Wood

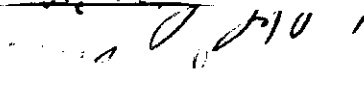
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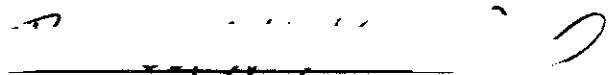
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SUMMARY

Previous work on Tm^{170} indicates an allowed shape and a log ft value of about 9 for the 883 keV beta group. A spin of one has been measured for the ground state of Tm^{170} , and the 883 keV beta transition is first forbidden with a spin change of one.

In the present investigation the transverse beta particle polarization, relative to the plane defined by the beta and subsequent 84 keV gamma ray, has been determined at an average beta particle energy of 266 keV. The experimental results have been compared with the theory of transverse beta particle polarizations. In the interpretation of beta decay experiments it is necessary to have data on several kinds of experiments (e.g. the half life, the shape of the beta spectrum, the end point energy, relevant polarizations, directional correlations, etc.) involving a single decay to determine or even severely delimit the nuclear matrix elements for that decay. The observables of first forbidden beta decay with a spin change of unity may be expressed in terms of four matrix elements.

The Tm^{170} was produced at the Oak Ridge National Laboratory by irradiation of Tm^{169} with thermal neutrons. The source was prepared by evaporation of TmCl_3 in HCl solution onto 0.3 mil aluminized mylar foil. The equipment used was standard scintillation spectrometers with a coincidence circuit permitting the study of coincidences between radiations of selected energies. The gamma spectrum exhibited a strong peak at 84 keV and the gamma detector was set on this peak throughout the experiment.

The beta detector was calibrated with the 364 keV conversion electron of Sn^{113} . The beta detector accepted beta particles in the energy range from 215 keV to 385 keV.

Mott scattering was used to detect the two components of the transverse polarization - perpendicular to the plane defined by the beta and gamma momenta and parallel to the plane defined by the beta and gamma momenta. A gold foil 1.8×10^{-5} inch thick was used to detect the asymmetric scattering due to the polarization; a second gold foil 10^{-4} inch thick was used to determine the experimental asymmetries. It was assumed that multiple scattering removed any polarization effects from the latter measurement. The angle between the beta and gamma momenta was fixed at 135° . The Mott scattering angle of the beta by the gold foil was about 120° .

The two polarizations were calculated from the scattered beam intensities at four counting positions and were corrected for depolarization in the scattering foil and for geometrical attenuation. The transverse polarizations were measured as

$$P_{\perp} = 0.00 \pm 0.03$$

$$P_{\parallel} = 0.05 \pm 0.03$$

where P_{\perp} is the perpendicular component and P_{\parallel} is the parallel component of the polarization.

In order to interpret theoretically the polarization data in terms of the nuclear matrix elements governing the beta decay, use was made of limitations placed on the matrix element parameters by previous experiments on the beta spectral shape and the beta-gamma directional

correlation. Using the accepted beta decay theory the expected beta transverse polarizations were calculated. These predicted polarizations were found to disagree in sign, but not in magnitude, with those measured. It is suggested that the theory be refined to accommodate the use of more exact electron radial wave functions. That there may be ambiguity in the comparison of experiment and theory regarding the sign of the polarizations is also noted.

CHAPTER I

INTRODUCTION

Purpose of this Research

The basic aim of this research is to use beta decay as a probe of nuclear structure. In a typical beta decay a neutron inside a nucleus decays into a proton with the simultaneous creation of an electron and an antineutrino. Even though the emission of beta particles was one of the first observed manifestations of radioactivity, significant theoretical developments and experiments which can yield information on nuclear structure and the interaction producing beta decay are of more recent origin. In 1934 Fermi proposed a beta decay theory which has served as the foundation of future development. Present work with the shell and unified models (1), (2), (3), (4), (5), (6) makes possible the fruitful interpretation of rather detailed types of experiments in addition to the traditional experiments on the gross nuclear properties such as spin, parity, and energy.

The experimental verification of the non-conservation of parity rekindled interest in beta decay. It is interesting to note that these developments were the consequences of interest in the strange particle interactions - decays which involve the "weak interaction," the interaction governing beta decay. As a consequence of this increased interest the form of the beta decay interaction is now well established (7).

In the interpretation of beta decay experiments it is necessary to have data on several kinds of experiments (e.g. the half life, the

shape of the beta spectrum, the end point energy, relevant polarizations, directional correlations, etc.) involving a single decay to measure or even severely delimit the nuclear matrix elements for that decay.

Allowed beta decay involves only two nuclear matrix elements, and many experiments have been performed to measure them for particular decays. The observables of first forbidden beta decay with a spin change of unity involve four matrix elements. The intent of the present work was to obtain transverse beta ray polarization data on the first forbidden beta decay of Thulium 170 for which additional types of data are known from previous experiments.

Choice of Thulium

Prior to commencement of this research a computer program was written for the Burroughs 220 at the Rich Electronic Computer Center for the purpose of estimating the expected transverse beta polarization in selected first forbidden decays. The formulas for the polarization given by Kotani (8) were evaluated from estimates of the matrix element parameters suggested by H. G. Dulaney (9). Weakly energy dependent values for the transverse polarization were obtained for various sets of matrix element parameters for each of the five nuclides considered: Sb^{124} , Eu^{152} , Eu^{154} , Tm^{170} , and Re^{186} . For Tm^{170} Dulaney's choice of possible parameters predicted polarizations ranging from one to seven percent, the largest value found for any of the nuclides considered. Since Tm^{170} has a favorable half life and a simple decay scheme, it was chosen as the object of this research.

Existing Knowledge of Tm^{170}

Tm^{170} decays to Yb^{170} by electron emission. The decay scheme is

shown in Figure 1 (10).

The ground state spin of Tm^{170} has been measured as one by magnetic resonance experiments (11). Internal conversion data indicate that the 84 keV gamma ray is E2 (12), which establishes the 84 keV level in Yb^{170} as 2^+ . The spin sequence is then $1^-(\beta)2^+(\gamma)0^+$.

Fermi plot analysis of the beta spectrum gives a log ft value of about 9.3 for the 883 keV beta transition with an allowed shape (13), (14), (15). The beta-gamma directional correlation has been measured by several experimental groups (15), (16), (17). H. G. Dulaney, C. H. Braden, and L. D. Wyly (18) give possible values of the matrix element parameters for Tm^{170} based on the measurements of the beta-gamma directional correlation and the beta spectral shape.

Determination of Beta Decay Matrix Elements

Apart from gross nuclear properties such as spin, parity, and energy level spacing, the effects of nuclear structure on beta decay are confined to the matrix elements (19), (20) of the decay. In principle these matrix elements can be determined by measuring various observables of the decay; however, the experimental results usually serve only to place certain limitations on the matrix elements, and it is only in a few cases that the values of all the individual matrix elements have been measured. (The quantity determined is, in fact, a matrix element multiplied by a coupling constant, and it is conceivable that the coupling constants for the decay of a free neutron differ somewhat from those of a neutron in the pion field inside a nucleus.)

In allowed beta decay the observables that give information about the matrix elements are the ft value and the directional correlation

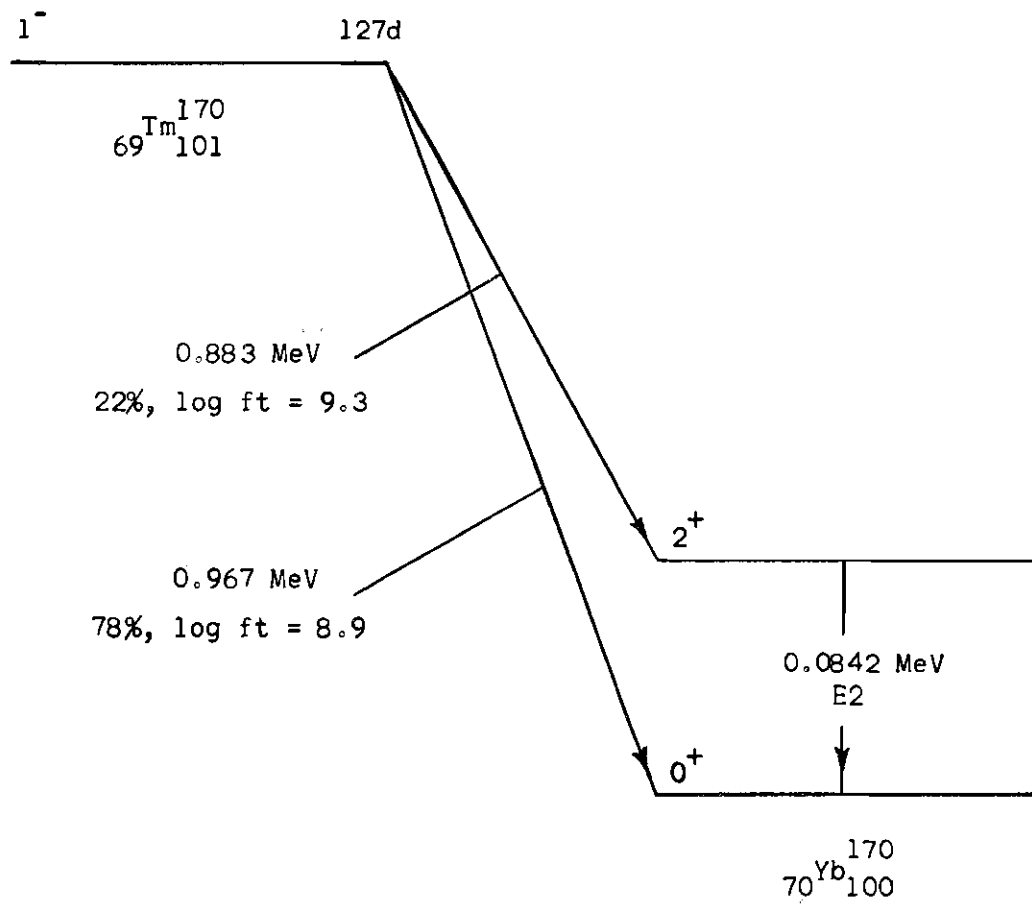


Figure 1. Decay Scheme of Tm^{170} .

between electron and neutrino momenta. The suggestion of Lee and Yang (21) that there was no evidence for conservation of parity in the weak interaction led to an extension of the Fermi theory and to the discovery of important new experimental facts. The experiment by Wu et al. (22) on the angular-distribution of the beta particles from the decay of oriented Co^{60} indicated that parity was not conserved. Parity non-conservation led to a revival of the two component neutrino which, along with other evidence such as the fact that electrons emitted have left-handed helicity, led to the acceptance of the V-A interaction Hamiltonian.

First forbidden beta decay yields much more information regarding matrix elements than the allowed decays. Observables which yield information regarding matrix elements in first forbidden beta decay include:

- (1) Shape correction factor,
- (2) ft value,
- (3) Beta ray angular distribution from oriented nuclei,
- (4) Beta-gamma directional correlation,
- (5) Beta-circularly polarized gamma correlation,
- (6) Longitudinal polarization of the beta,
- (7) Longitudinally polarized beta-gamma correlation,
- (8) Transversely polarized beta-gamma correlation,
- (9) Beta-conversion electron polarization correlation,
- (10) Positron to K-capture ratio.

The shape correction factor is an energy dependent function which must be applied to the Fermi-Kurie plot of the beta spectrum to make it a straight line. For methods used in obtaining the factor, see Graham, Wolfson, and Bell (23). The ft value involves the absolute

determination of the decay rate and can be obtained by methods used in G. Bertolini et al. (15). The beta ray angular distribution from oriented nuclei was first measured by Wu et al. (22). The beta-gamma directional correlation requires detection of the beta and the subsequent gamma in cascade from the same decaying nucleus. Methods of making this measurement are illustrated by Dulaney, Braden, Patronis, and Wyly (17). The beta-circularly polarized gamma correlation requires measurement of the gamma polarization, e.g., by Compton scattering in the experiment of R. M. Steffen (24). The longitudinal polarization of the beta particles may be detected by precessing their spins to an orientation perpendicular to their momenta and observing an asymmetric Mott scattering of the electrons as in A. R. Brosi et al. (25) or by the use of Møller scattering as done by H. Frauenfelder et al. (26). The longitudinally polarized beta-gamma correlation requires coincidence detection of the beta longitudinal polarization and the following gamma. The transversely polarized beta-gamma correlation, the subject of this thesis, requires a measurement of the transverse polarization of the beta, which may be determined by Mott scattering as in the experiment of Simms and Steffen (27). The beta-conversion electron polarization correlation is a consequence of the gamma circular polarization and has been observed by B. Blake et al. (28). Positron to K-capture ratio is the ratio of the respective decay constants for these two modes of decay. The ratios have been tabulated by Feenberg and Trigg (29).

Morita and Morita (30) and Kotani and Ross (31) give formulas for some of the observables in terms of the matrix elements. Using the Kotani and Ross formulas and results of previous experiments, H. G. Dulaney,

C. H. Braden, and L. D. Wyly (18) give possible values of the matrix element parameters for beta decays in Re^{188} , Re^{186} , Tm^{170} , Eu^{154} , Eu^{152} , and Sb^{124} .

For a first forbidden beta decay with spin change of one the relevant nuclear matrix elements are generally denoted as (32):

$$\int \bar{r}, \int \bar{a}, \int \bar{\sigma} \times \bar{r}, \int B_{ij}.$$

There have been several estimates of the relative size of the $\int \bar{r}$ matrix element and the $\int \bar{a}$ matrix element based on theoretical arguments. These estimates are generally expressed in terms of the parameter

$$\Lambda = i \int \bar{a} / \xi \int \bar{r}$$

where ξ is a dimensionless expansion parameter defined by $\xi = \alpha Z / 2\rho$ where Z is the atomic number of the daughter nucleus, ρ is the nuclear radius, and $\alpha = 1/137$ (the fine structure constant). For Tm^{170} $\xi = 14.8$. A theoretical estimate for Λ was first made by Ahrens and Feenberg (33) using the single particle model and uniform nuclear density. Pursey (34) computed a different numerical value. Recently the conserved vector current theory of weak interactions was used by Fujita (35) and independently by Eichler (36) to derive a theoretical estimate for Λ . The proposed theoretical values for Λ may be summarized as follows:

$$\Lambda = 1.0 + (W_i - W_f) A^{1/3} / Z, \quad \text{Ahrens and Feenberg,}$$

$$\Lambda = 2.0 + (W_i - W_f) A^{1/3} / Z, \quad \text{Pursey,}$$

$$\Lambda = 2.4 + (W_i - W_f) A^{1/3} / Z, \quad \text{Fujita (CVC Theory).}$$

Here, $W_i - W_f = W_o - 2.5 mc^2$ for electron emission, where W_o is the total end point energy of the betas, A is the mass number, Z the atomic number of the daughter, and mc^2 is the electron rest energy. Generally, $(W_i - W_f) A^{1/3}/Z$ will be of order 0.1. Therefore the Ahrens and Feenberg relationship and the Fujita relationship may be characterized by $\lambda = 1.0$ to 1.2 and $\lambda = 2.4$ to 2.6 , respectively.

There has also been considerable interest in the size of the B_{ij} matrix element as compared to other first forbidden matrix elements. Morita and Morita (30) found that the experimental results for several first forbidden beta decays could be satisfactorily analyzed by assuming the B_{ij} matrix element to be much larger than the other first forbidden matrix elements. This assumption is generally referred to as the "modified B_{ij} approximation." Kotani (8) has suggested a theoretical basis for the modified B_{ij} approximation. He has proposed that the attenuation of the other matrix elements, relative to the B_{ij} matrix element, may be due to selection rules involving the concepts of "K forbiddenness" and "j forbiddenness."

Dulaney et al. (18) and Bogdan (5), (6) discuss these theoretical estimates in relation to experimental results.

Brief History of Polarization Detection

One may think of producing a polarized beam of electrons by means of a kind of "Stern-Gerlach" experiment, i.e., sending electrons through a strongly inhomogeneous magnetic field to separate the two spin states; however, in a well-known argument Bohr and Mott (37) show that a splitting of an electron beam according to spin orientation cannot be obtained in this way. The inhomogeneity of the magnetic field causes a spreading of

the charged electron beam (the particles of a proper Stern-Gerlach experiment are electrically neutral) which is so large that the spreading arising from different orientations of the magnetic moment in the inhomogeneous magnetic field does not lead to two distinct beams of electrons.

In analogy with the polarization of light by reflection at a mirror, one may think further of the possibility of obtaining polarized electrons by reflection of a beam by a sudden change of potential (Malus effect). A discussion of this possibility in a number of theoretical papers led to a simple proof that it is also impossible to obtain electron polarization in this way (38), (39). The attainment or detection of electron polarization that was based on the use of macroscopic electric or magnetic fields proved fruitless.

The most commonly used method of measuring polarization is based on scattering of electrons by heavy nuclei - Mott scattering. Consider an electron approaching a nucleus of charge Ze as in Figure 2. For case (a) $\vec{\sigma} \cdot \vec{L}$ is greater than zero; for case (b) $\vec{\sigma} \cdot \vec{L}$ is less than zero, where $\vec{\sigma}$ is the spin of the electron and \vec{L} is the orbital angular momentum of the electron with respect to the target nucleus. It is obvious that the spin-orbit term, $\vec{\sigma} \cdot \vec{L}$, contributes to the interaction Hamiltonian in a fashion that depends upon the orientation of the electron spin. Mott has solved the Dirac equation for an electron in a central field to calculate the effect of electron polarization on scattering (40), (41). Appendix A contains an outline of the quantitative character of the polarization effect in Mott scattering. If a beam of electrons traveling in the z direction is incident on a scattering center at the origin and is scattered into the yz -plane, then equation (A-17) states

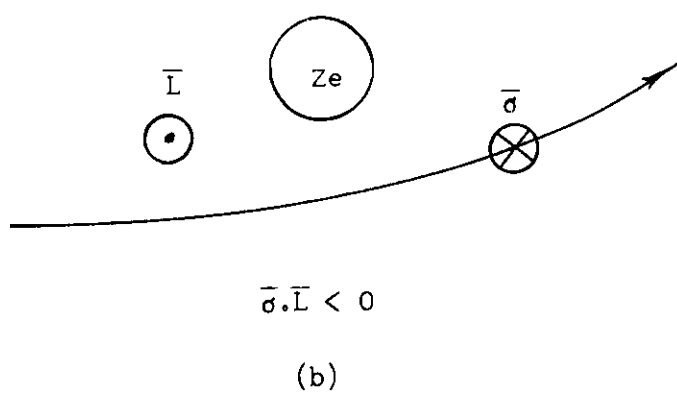
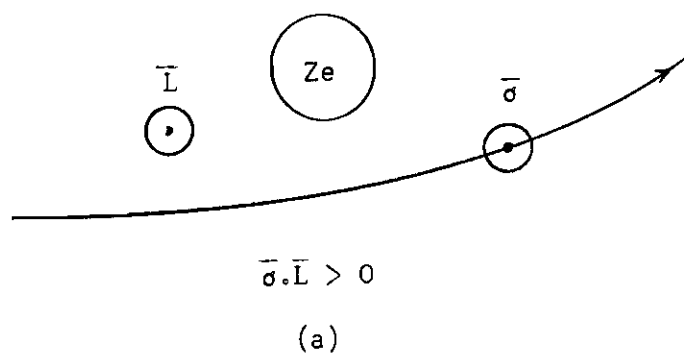


Figure 2. Electron Scattering.

$$I(\theta, y)/I(\theta, -y) = 1 - 2S(\theta)P_x \quad (1-1)$$

where $I(\theta, y)$ is the intensity of the scattered beam with scattering angle θ and positive y component of momentum after scattering, $S(\theta)$ is Sherman's function (See Appendix A, equation (A-16)), and P_x is the x component of the electron beam polarization.

One may also compute the polarization produced in the electron beam if an unpolarized beam is scattered from a foil. Then one may analyze the scattered beam and detect the polarization with another foil (40), (41) - the double scattering experiment.

Until 1942 experimental results on double electron scattering failed to show any polarization effect. Careful experiments by E. G. Dymond (42), (43) and G. P. Thomson (44) gave negative results. The experiments were done mainly with gold foils because of the high atomic number and easy fabrication. These foils must be exceedingly thin in order to assure that the detected electrons have suffered only a single elastic collision. A depolarization may originate in the following ways: (a) inelastic scattering with ionization or excitation of the atom; (b) exchange scattering; if the incident electron exchanges positions with an atomic electron, the polarization will be lost; (c) multiple scattering; if the final scattering angle is obtained by a succession of small angle scatterings almost no polarization effect exists; (d) plural scattering; we shall speak of plural scattering if the final scattering angle is the result of two scatterings over rather large angles. The contribution by plural scattering to the total scattering is appreciable if the first scattering is into the plane of the foil and if the two scattering angles are both smaller than the resultant scattering angle.

(The differential scattering cross section increases rapidly as the scattering angle decreases.) The influence of the causes of depolarization have been discussed by Rose and Bethe (45). H. Wegener (46), (47) has calculated the attenuation of the polarization as a function of foil thickness. It follows that scattering foils of thickness the order of 10^{-5} cm., as were used in the above-mentioned experiments, are thin enough for the first three causes of depolarization to be negligible. The negative results are considered to be a consequence of plural scattering. The first successful double scattering experiment was performed by Shull, Chase and Myers (48) using a gold foil 4.1×10^{-5} cm. thick in an improved geometry, the "transmission" position. The transmission position reduces the attenuation due to plural scattering. A comprehensive review of polarization detection prior to 1956 was made by H. A. Tolhoek (49).

In 1956 Lee and Yang (21) proposed that parity might not be conserved in weak interactions and suggested various experiments to test their hypothesis. After two successful experiments - the asymmetry of the angular distribution of electron emission from aligned nuclei (22) and the polarization of muons (50) - Lee and Yang discussed a two-component neutrino theory (51) and suggested the simultaneous measurement of momentum and polarization of electrons emitted in beta decay. If parity were not conserved, the electrons should be longitudinally polarized. Frauenthal et al. (52) measured the longitudinal polarization of electrons from Co^{60} to be approximately $-v/c$ where v is the speed of the electron and c is the speed of light. By means of an electrostatic deflector, the longitudinal polarization was changed into a transverse one which was

measured by the Mott scattering technique. Cavanagh et al. (53) performed an equivalent experiment and found similar results. Since then much work has been done with longitudinal polarizations. A. R. Brosi et al. (25) have made a very precise determination of the longitudinal polarization of the beta particles emitted in the allowed decay of P^{32} .

Rose and Becker (54) have predicted that internal conversion electrons following beta transitions should possess an appreciable transverse polarization. Such polarizations have been measured by Blake et al. (55), Vishnevskii et al. (56), and Alberghini and Steffen (57).

Curtis and Lewis (58) proposed the observation of the transverse beta polarization in a beta-gamma directional correlation experiment. Simms and Steffen (27) looked for the effect in the first forbidden beta decay of Au^{198} . They found a polarization of 0.011 ± 0.005 .

CHAPTER II

EXPERIMENTAL PROCEDURE

Instrumentation

Electronics

The electronic equipment used in this experiment consisted of two scintillation spectrometers with a coincidence circuit permitting the study of coincidences between radiations of selected energies. A block diagram of the circuits is shown in Figure 3.

The beta detector used a 1-1/2 inch diameter by 0.3 cm. high Spectrum Plastic Scintillator supplied by Semi-Elements, Incorporated, Saxonburg, Pennsylvania. The scintillator thickness is about the minimum thickness required for total absorption of electrons in the energy range of interest. The scintillator was coupled through a lucite light pipe to a Radio Corporation of America number 6810A photomultiplier tube. The gamma detector used a 1-1/2 inch diameter by one inch high NaI(Tl) crystal coupled to a Radio Corporation of America number 6810A photomultiplier tube. The photomultiplier cathode was shielded against magnetic fields which might have influenced the counting efficiency of the detector. NJE Model S325 power supplies were used to supply the high voltage for the photomultiplier tubes. The beta tube was operated at 1750 volts; the gamma tube was operated at 2000 volts. The detectors were mounted to chassis which incorporated voltage divider networks to furnish the correct potentials to the various photomultiplier electrodes, cathode follower stages to pick off the pulses that were then sent to the

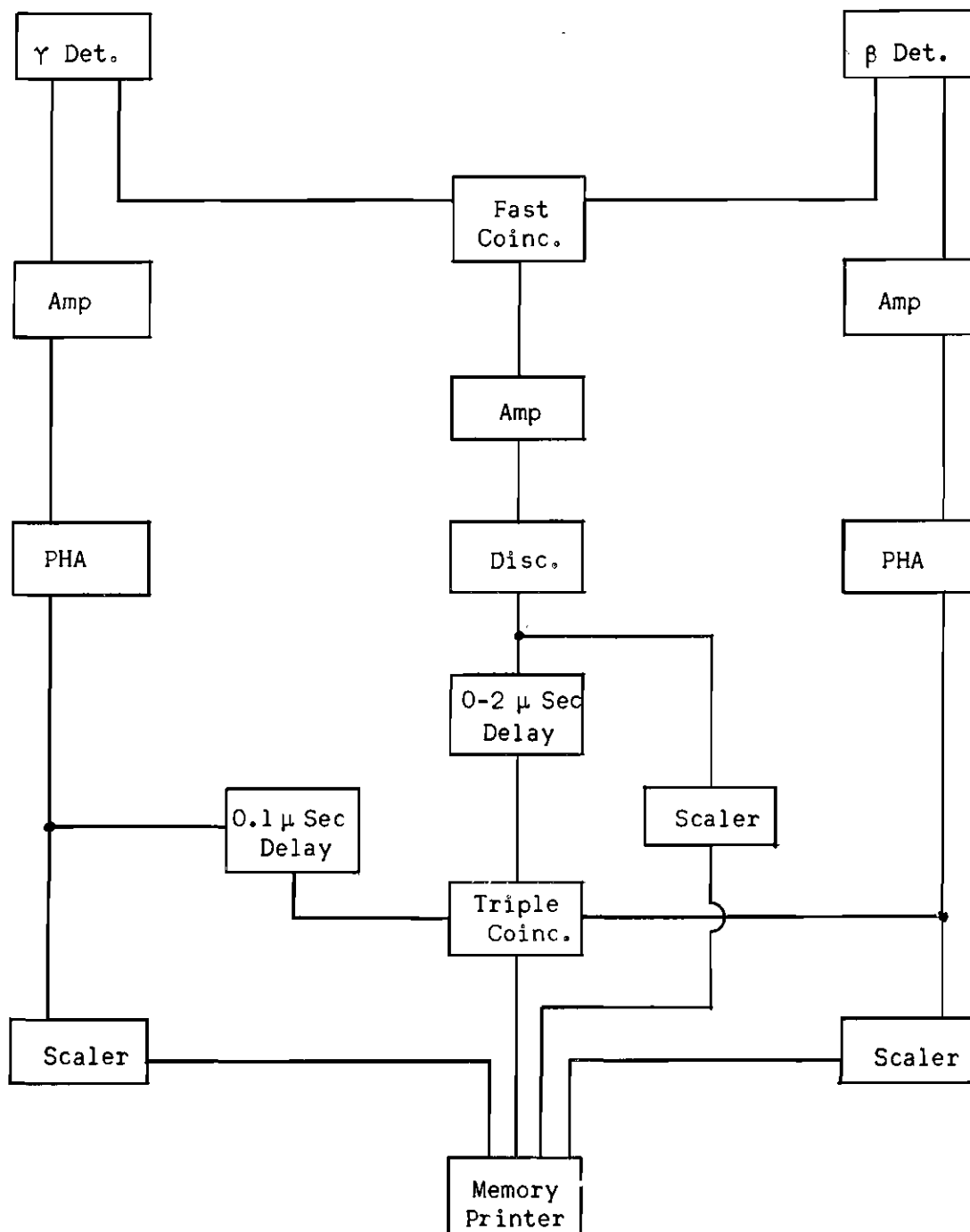


Figure 3. Block Diagram of Electronics.

amplifiers and pulse height analyzers, and limiter tubes for the fast coincidence circuit.

The following additional electronic equipment was used: Hamner Model N301 amplifier in the fast circuit, Hamner Model N302 amplifier and pulse height analyzer in the beta channel, a double delay line amplifier designed by Dr. E. T. Patronis in the gamma channel, Radiation Counter Laboratories Model 2204 pulse height analyzer in the gamma channel, Hamner Model N220 scaler in the gamma channel, Atomic Instrument Company Model 1020 scalars in the gamma channel and the fast coincidence circuit, Radiation Counter Laboratories Model 2006 scaler in the beta channel, and a triple coincidence circuit designed by Dr. E. T. Patronis.

The fast coincidence circuit is, in principle, that of Bell and Petch (59). The circuit diagram is given in Figure 4. Signal sources for Western Electric type 404A pentode limiters are the two type 6810A 14-stage photomultiplier tubes. The limiters are located adjacent to the photomultiplier tube anodes. After the limited pulses have been "clipped" by action of the shorted section of RG62U cable, they are 0.9 volt high and have a half width of 20 nanoseconds. The pulse heights are matched at the diode by varying the plate voltages of the limiters. The diode is biased at about one volt. The output of the coincidence circuit is fed into an amplifier at the output of which the "capacitive feed through" pulses are less than two volts, and the coincidence pulses are as high as 125 volts. (Capacitive feed through is the result of capacitance inherent in the construction of a diode.) The amplifier output goes to a discriminator which passes pulses higher than about 70 volts. The discriminator level permits an adjustment of the fast coincidence

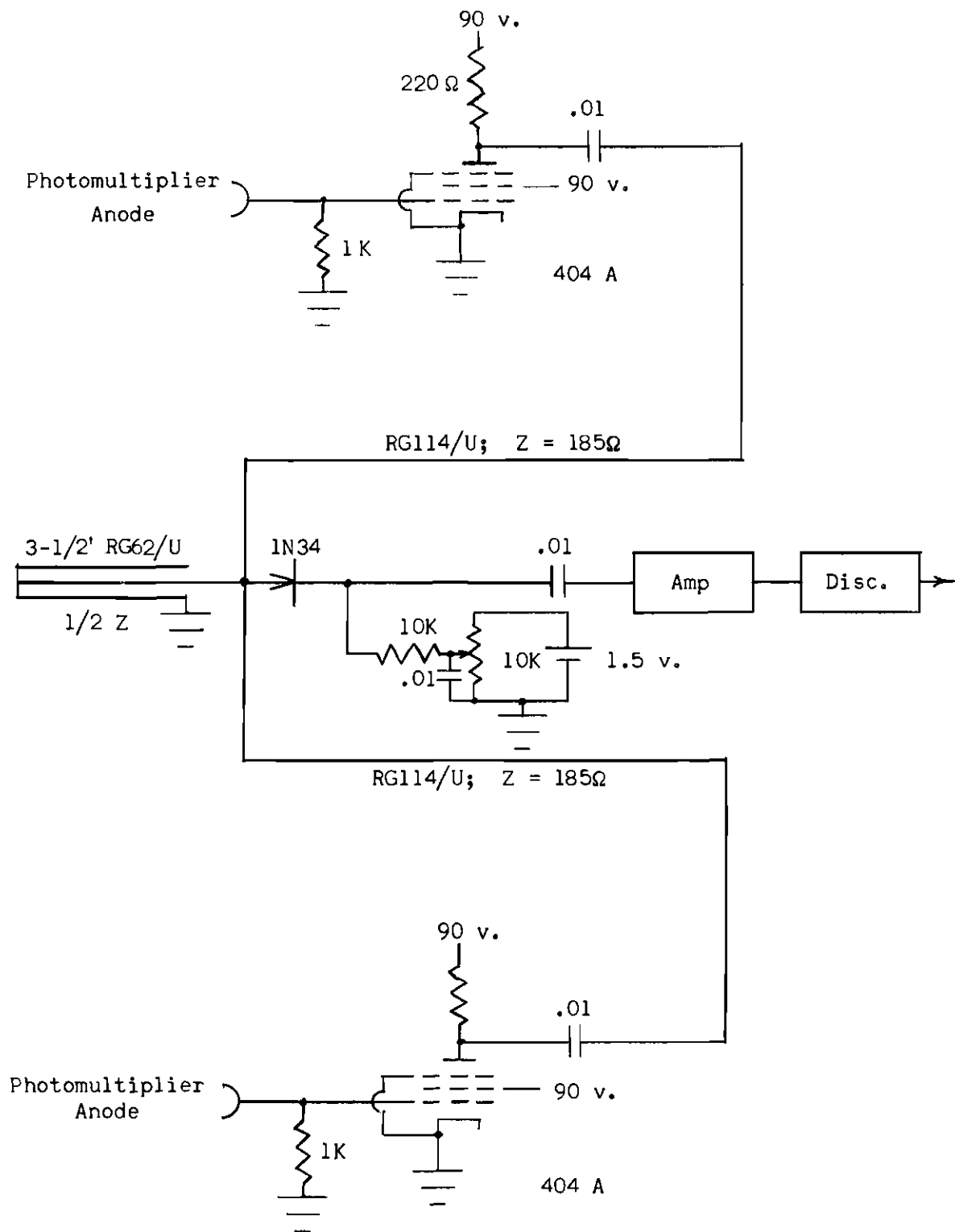


Figure 4. Diagram of Coincidence Circuit.

resolving time. The output of the discriminator is fed into the triple coincidence circuit which has a resolving time of about 0.5 microsecond. The circuit operated reliably for months at a resolving time of six nanoseconds.

The memory-printer-control unit was designed and built by Mr. N. S. Kendrick. Western Electric furnished the selector switches necessary for the construction of the unit through their college gift program.

Counting Geometry

A photograph of part of the apparatus is given in Figure 5, and a diagram of the geometry is given in Figure 6. The cylindrical aluminum vacuum chamber measures 12 inches inside diameter by 24 inches high. The beta counter enters the chamber through a hole in the side, so the axis of the beta counter is perpendicular to that of the chamber. Beta particles enter the chamber at the top. Since the inside of the scattering chamber is in a high gamma ray flux, the metal surfaces, which might emit Compton electrons, were covered with a 1/2-inch thick sheet of lucite. (Five sixteenths inch of lucite will stop a two MeV beta particle.) The chamber depth was chosen as two feet so that at the gold scattering foil the beta ray flux downward should be at least 1000 times the upward flux. Enough lead shielded the beta counter from direct gamma rays to assure an intensity attenuation of 10^{-4} for one MeV gammas.

The Mott scattering foils were mounted in a plane perpendicular to the chamber axis on lucite rings of 5-1/2 inch inside diameter. The 10^{-4} inch thick gold foil was made by A. D. MacKay, Incorporated, New York. The 1.8×10^{-5} inch thick gold foil was gold leaf manufactured by Hastings and Company, Incorporated, Philadelphia, Pennsylvania. The gold

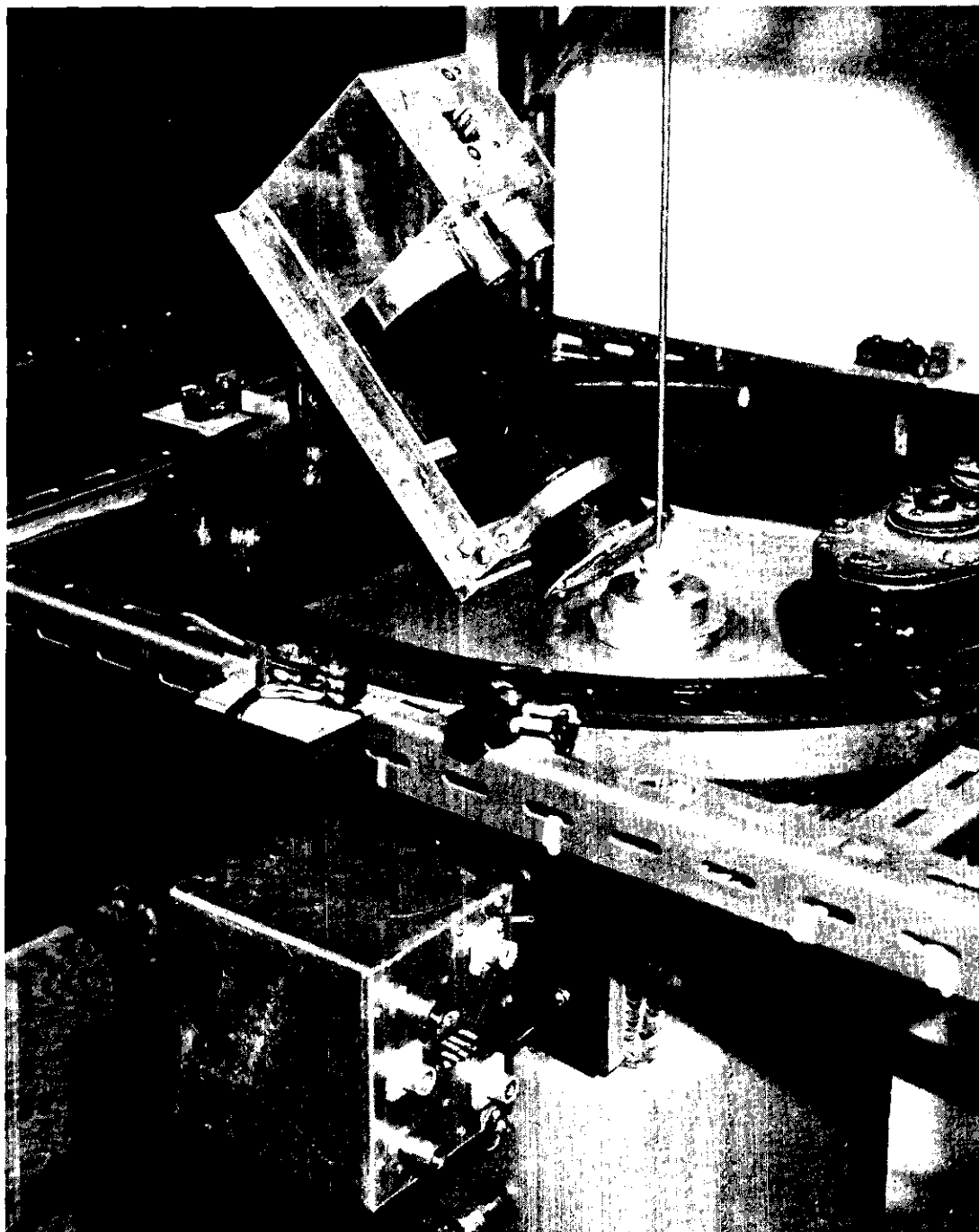


Figure 5. (See next page.)

Figure 5. Photograph of Vacuum Chamber and Detectors. The photograph shows the scattering chamber and the beta counter with attached preamp in the left foreground. The gamma counter and preamp are at the upper left in the geometry described as the 90 degree position. The source is inside the thin walled aluminum cover on top of the chamber and immediately in front of the gamma counter face. The torsion rod for the source rotator is also visible above the source cover.

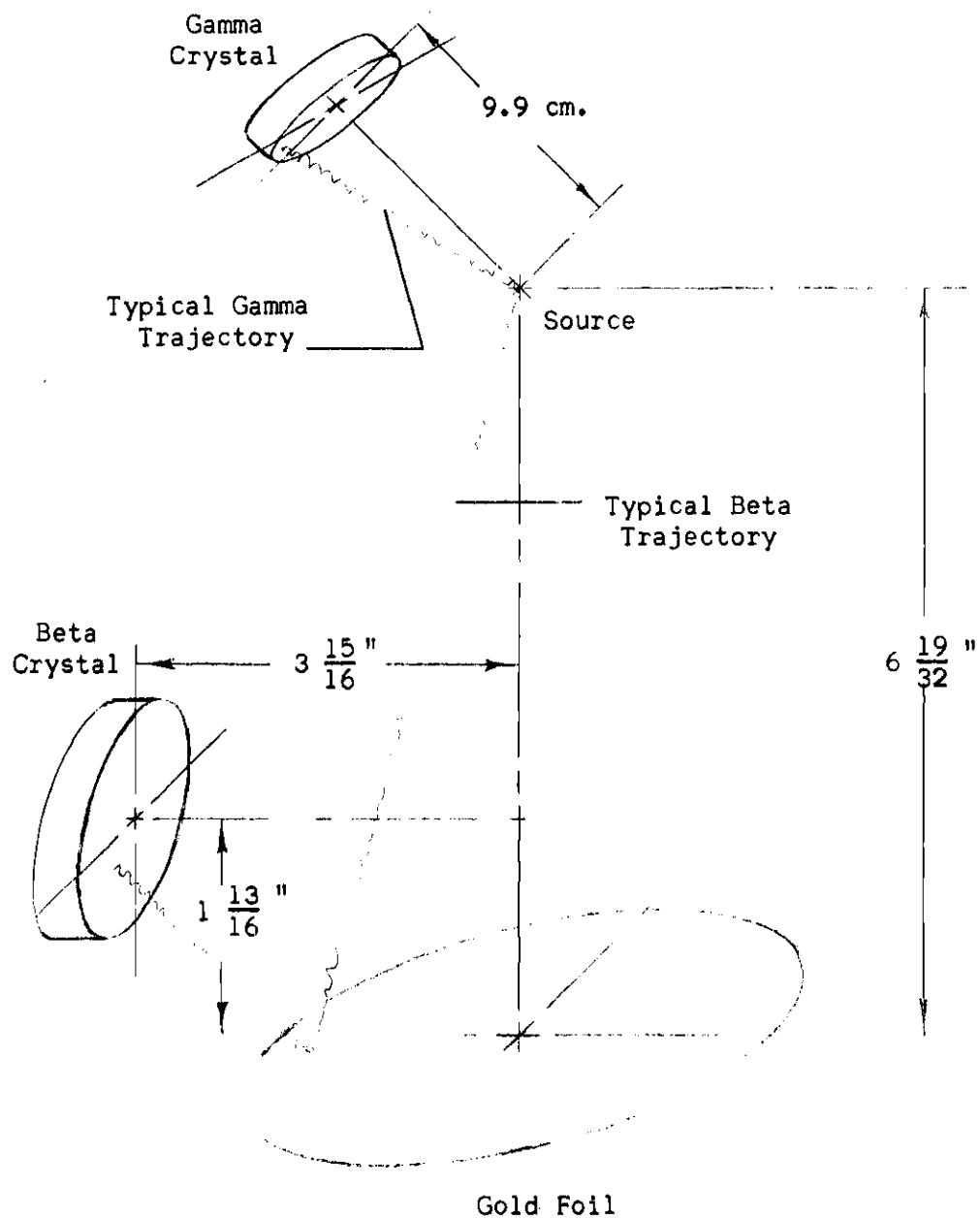


Figure 6. Counting Geometry.

leaf was supported by a grid of nylon filaments stretched across the support ring at 1/2 inch intervals.

Sherman's function (defined in Appendix A) is analogous to an amplitude of the polarization effect. Upon examination of his table (60) for the function it is found to be a maximum in the neighborhood of 120 degrees for the beta scattering angle throughout the range of beta particle energies of interest (50 keV to 600 keV). Accordingly, the geometry was arranged to secure a scattering angle in the range 110 degrees to 130 degrees.

The source was placed on the axis of the chamber at its top, and the beta rays were defined into a cone (apex up, axis vertical) having apex angle of 30 degrees by a double baffle lucite collimator. The source was rotated at one revolution per minute about a vertical axis to reduce instrumental asymmetries.

The source, Mott scattering foil, and beta crystal were inside the vacuum chamber which was maintained at a pressure of ten microns or less. An aluminum cap having a wall thickness of 0.01 inch covered the source and allowed the gamma rays to pass out of the vacuum chamber and into the gamma counter whose axis passed through the source and made a 135 degree angle with the downward, vertical chamber axis. An "O" ring seal in the top of the cap allowed the mechanical rotation of the source.

The 135 degree angle between momenta of the beta and gamma rays was chosen for this experiment in order to maximize the polarization of the beta particles. The theoretical predictions of Kotani and Ross (31) for the polarization contain the factor $\sin \theta \cos \theta$ where θ is the angle subtended at the nucleus by the beta and gamma ray momenta.

The gamma counter was periodically rotated about the chamber axis in 90 degree increments. The momenta of the emitted beta and gamma rays define a plane. Consider a rectangular cartesian coordinate system as depicted in Figure 7. The origin is at the source and the incident beta ray momentum defines the z axis. The xz -plane is defined as that plane which contains the momenta of the beta incident on the scattering foil and the gamma. We will speak of the zero position when the beta counter is in the xz -plane with positive x coordinate; the 90 degree position when it is in the yz -plane with negative y coordinate, the 180 degree position when it is in the xz -plane with negative x coordinate; and the 270 degree position when it is in the yz -plane with positive y coordinate. In this fashion the ratio of scattered beta intensities at zero degrees and 180 degrees is a measure of the beta particle transverse polarization perpendicular to the plane of the beta and gamma, and the ratio of scattered intensities at 90 degrees and 270 degrees is a measure of the transverse polarization in the plane of the beta and gamma rays. Note that the beta counter is fixed during the experiment and the angle is altered by rotation of the gamma counter which thereby rotates the xz -plane. The angle between the beta and gamma momenta is defined to be 135 degrees (as opposed to 225 degrees) which then defines a right handed coordinate system that should be consistent with that employed in the theoretical analysis of T. Kotani (8), (31).

Source Preparation

The Tm^{170} was produced at the Oak Ridge National Laboratory by irradiation of Tm^{169} with thermal neutrons. The source was prepared by evaporation of TmCl_3 in HCl solution onto 0.3 mil aluminized mylar foil.

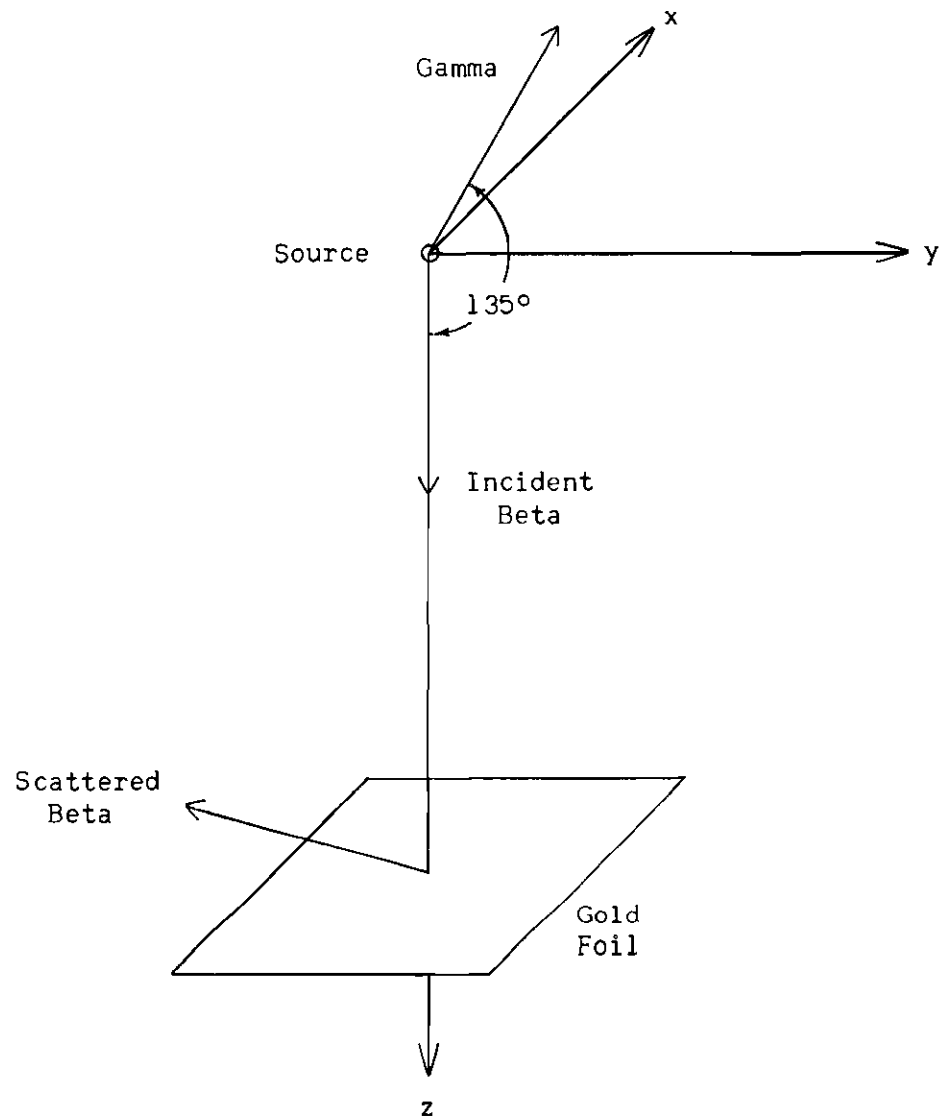


Figure 7. Coordinate System.

The source was about 0.3 cm. in diameter. The mylar was glued between two similar lucite rings. One was concentric with the collimator; the other was covered with 1/4 mil mylar to incapsulate the source. This arrangement required the beta particles to pass through the aluminized mylar prior to entering the scattering chamber. The aluminized mylar touched the aluminum chamber thereby grounding the source. The initial source strength was estimated as 5 millicuries.

Preliminary Experiments and Considerations

Determination of the Gamma Singles Spectrum

A plot of the gamma spectrum of Yb^{170} is shown in Figure 8. The data for this plot were obtained with a window width of 0.1 volt on the pulse height analyzer. The 84 keV peak is the gamma peak to be used in this experiment; it appears at 8.8 volts. The 59 keV peak is the Yb^{170} X-ray which arises from internal conversion. Since the X-ray would only serve to flood the fast coincidence circuit and overload the singles amplifier, a 0.01 inch thick sheet of lead was placed in front of the gamma counter which favorably modified the gamma singles spectrum as can be seen in Figure 8. The spectrum indicates that a window width of two to three volts is a reasonable setting for the gamma pulse height analyzer during the polarization experiment.

Determination of the Beta Singles Spectrum

A plot of the beta singles spectrum of Tm^{170} is shown in Figure 9. The data for this plot were obtained with a window width of 0.1 volt on the pulse height analyzer. A beta energy of 364 keV corresponds to 10.9 volts. Also shown in Figure 9 are the beta singles spectra after scattering by a 1.8×10^{-5} inch and a 0.1 mil thick gold foil. The inverse

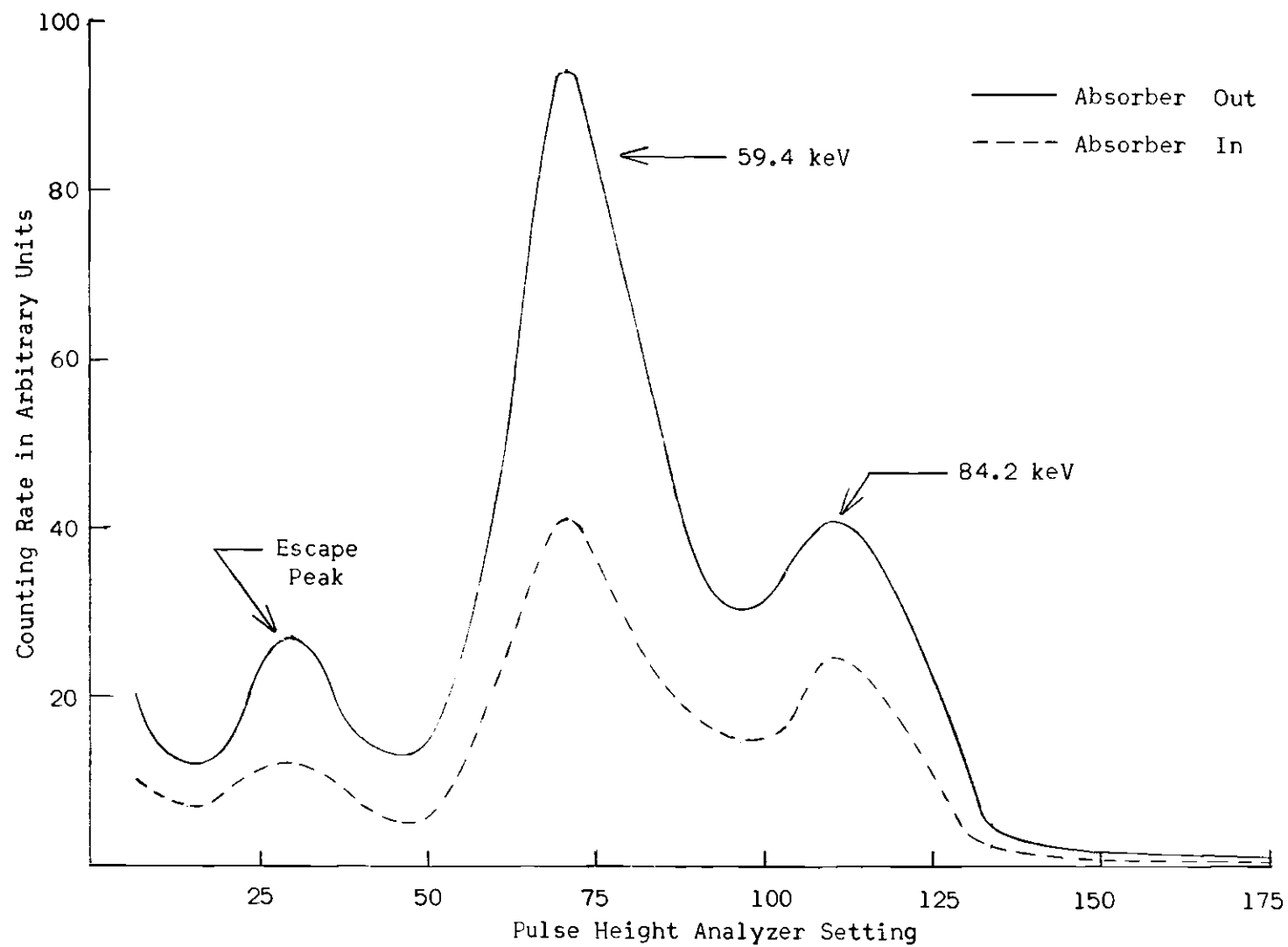


Figure 8. Gamma Ray Singles Spectra at Yb^{170} .

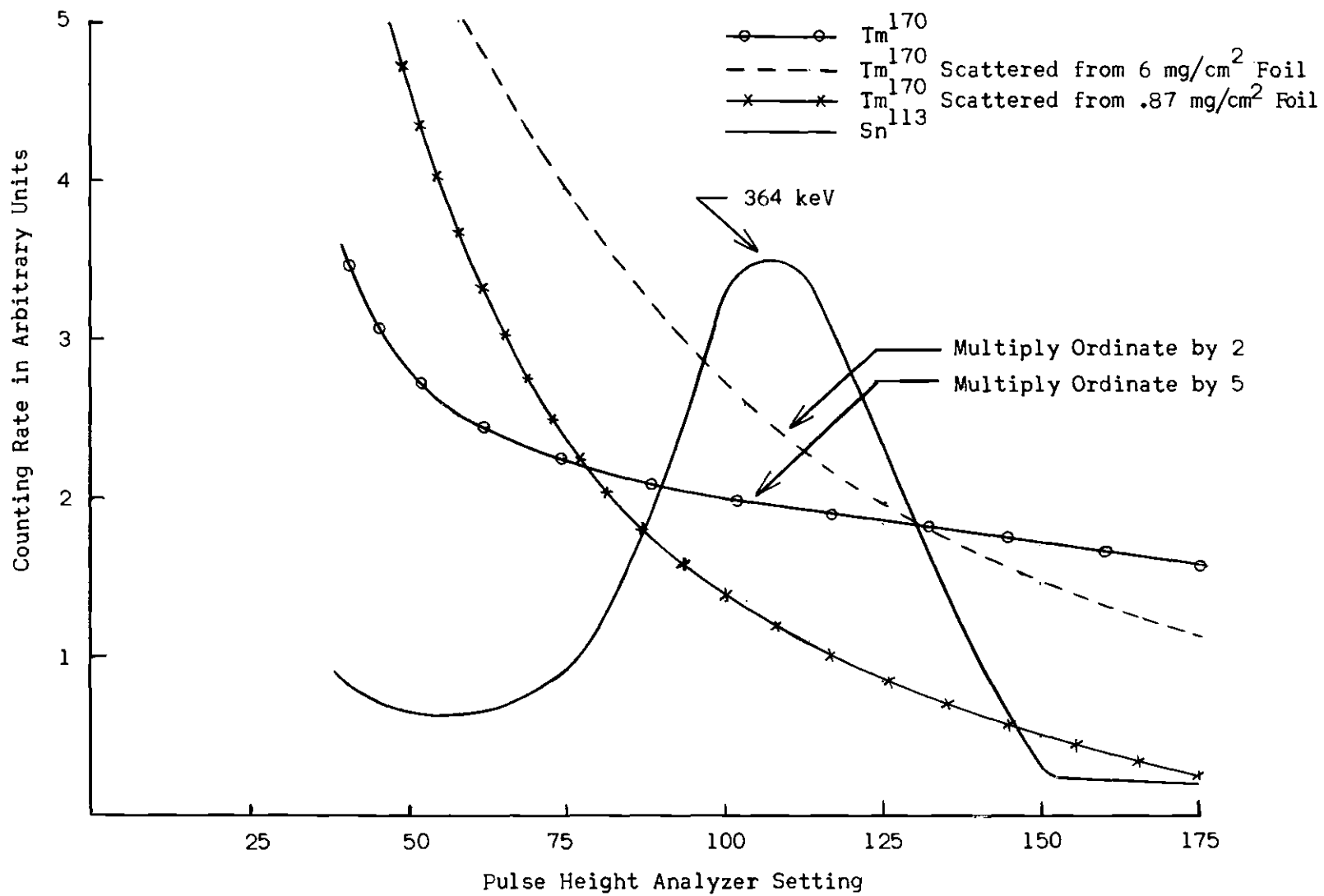


Figure 9. Beta Ray Singles Spectra of Tm^{170} and Sn^{113} .

dependence upon energy of the scattering cross section is evident in this figure. The probability of detecting coincidences between a beta ray and an X-ray is very low because of the energy resolution of the gamma detector.

Since there are no well defined peaks in the Tm^{170} beta spectrum, the beta detector must be calibrated with a different source. The region of interest is in the neighborhood of 300 keV. Tin 113 has a sharp conversion electron peak at 364 keV and makes a suitable calibration source. The spectrum of Sn^{113} is also shown in Figure 9. By adjusting the amplifier gain so that the conversion electron peak is at 10.9 volts, the pulse height analyzer is calibrated so that one volt corresponds to 33.3 keV. For the polarization experiment the pulse height analyzer was adjusted to accept pulses greater than 6.5 volts but less than 11.5 volts corresponding to acceptance of betas in the energy interval between 217 keV and 383 keV. The beta-gamma coincidence rate was then about 100 per hour.

Time Alignment of the Coincidence Circuits

The RG114U cables, that conduct the pulses from the beta and gamma detectors to the fast coincidence circuit, have a transmission delay of 1.2025×10^{-9} second per foot of cable. Cables were made which allowed time alignment of the fast coincidence circuit by simply placing the proper lengths in the two channels. The circuit produced the maximum coincidence rate when the beta pulses were delayed 2.1 nanoseconds more than the gamma pulses. This may be understood in terms of the longer decay time of the NaI phosphor as compared with the Spectrum Plastic Scintillator.

The slow coincidence circuit was aligned by reducing the bias on the diode in the fast coincidence circuit (depicted in Figure 4) so that singles pulses passed through the fast circuit. The "lumped constant" delay lines shown in Figure 3 were then varied for maximum triple coincidence counting rate. This alignment was necessary because delays of the order of microseconds are introduced by the amplifiers and pulse height analyzers.

Polarization Measurements

The transverse polarization may be found by use of equation (A-17) from Appendix A:

$$I(\theta, \varphi)/I(\theta, \varphi + \pi) = 1 - 2S(\theta)P(\varphi) . \quad (2-1)$$

Here θ is the scattering angle, φ is measured azimuthally from some direction perpendicular to the incident beam of beta particles, $I(\theta, \varphi)$ is the intensity of the scattered beam in a direction defined by θ and φ , $S(\theta)$ is Sherman's function defined in Appendix A, and $P(\varphi)$ is the beta particle transverse polarization in a direction perpendicular to the direction φ . The transverse polarization perpendicular to the plane defined by the beta and gamma rays, P_{\perp} , is given by

$$I(180^\circ)/I(0^\circ) = 1 - 2SP_{\perp} . \quad (2-2)$$

The transverse polarization in the plane of the beta and gamma rays, P_{\parallel} , is given by

$$I(270^\circ)/I(90^\circ) = 1 - 2SP_{\parallel} .$$

In this experiment the data obtained are the coincidence counting rates,

$I(0^\circ)$, $I(90^\circ)$, $I(180^\circ)$, and $I(270^\circ)$, at the four angles for which data were taken.

The following is a brief outline of corrections which had to be made to the coincidence data and the procedure that was followed in order to calculate the transverse polarizations from the coincidence data.

1. Normalization of the Beta-gamma Coincidence Rate

After the number of beta-gamma coincidences during an interval was determined the number was normalized by dividing it by the gamma singles counts during the interval. This normalization was necessary to correct for possible differences in the solid angle subtended by the gamma counter at different positions of the gamma counter. In Table 1 is given a sample of the data and the results of this normalization.

2. Correction for Accidental and Extraneous Coincidences

In order to obtain the number of real, beta-gamma coincidences for a given counting interval the number of accidental and extraneous coincidences was subtracted from the observed number of coincidences. The accidental coincidence rate was determined by inserting an 18 nano-second delay in one of the signal inputs to the fast coincidence circuit. This procedure was checked by the use of two independent sources and found to be acceptable. The extraneous coincidence rate was obtained by removing the scattering foil from the chamber and leaving all supporting mechanisms in place. This rate was found to be isotropic and about one percent of the real rate; it was neglected in the analysis. Examples of these corrections are given in Table 1.

3. Determination of the Average Beta Particle Energy

Since Sherman's function is dependent upon the beta particle energy,

Table 1. Sample of Data and Analysis for the Transverse Polarization Experiment

β Singles Counts $\div 1024; N_1$	γ Singles Counts $\div 256,000; N_2$	Total Coinc. N_c	Angle (degrees)	$N_c/N_2 \times 10^3$	$N_a/N_2 \times 10^3$	$(N_c/N_2 - N_a/N_2) \times 10^3$	Delay (nano- seconds)
253	166	97	0	584		394	2.1
252	157	90	90	573		383	2.1
251	155	84	180	542		352	2.1
254	161	79	270	491		301	2.1
251	166	85	0	512		322	2.1
252	163	98	90	601		411	2.1
252	162	81	180	500		310	2.1
251	162	95	270	586		396	2.1
252	165	84	0	509		319	2.1
253	164	94	90	573		383	2.1
253	161	105	180	652		462	2.1
253	162	86	270	531		341	2.1
253	163	97	0	595		405	2.1
254	163	93	90	571		381	2.1
252	163	96	180	589		399	2.1
253	162	89	270	549		359	2.1
250	163	82	0	503		313	2.1
254	163	101	90	620		430	2.1
252	160	93	180	581		391	2.1
252	162	91	270	562		372	2.1
256	165	29	0		176		18
254	160	34	90		213		18
250	160	29	180		181		18
252	162	31	270		191		18

$$\langle (N_c/N_2 - N_a/N_2)_{0^\circ} \rangle = 351.6$$

$$\langle (N_c/N_2 - N_a/N_2)_{180^\circ} \rangle = 382.8$$

$$I(180^\circ)/I(0^\circ) = 382.8/351.6 = 1.092$$

$$\langle (N_c/N_2 - N_a/N_2)_{90^\circ} \rangle = 397.6$$

$$\langle (N_c/N_2 - N_a/N_2)_{270^\circ} \rangle = 353.8$$

$$I(270^\circ)/I(90^\circ) = 353.8/397.6 = 0.890$$

a knowledge of the average energy of the particles that entered the beta counter was necessary. Using the scattered beta spectrum in Figure 9 an empirical weighting factor, dN/dE , was determined by fitting an exponential to the scattered beta particle spectrum in the energy range accepted by the beta counter, 217 keV to 383 keV. The average value of the energy was then determined by means of

$$\langle E \rangle = \frac{\int_{E_1}^{E_2} E \frac{dN}{dE} dE}{\int_{E_1}^{E_2} \frac{dN}{dE} dE} \quad (2-4)$$

where E is the beta particle energy, $E_1 = 217$ keV, and $E_2 = 383$ keV. The average beta energy found in this fashion was 266 keV.

4. Determination of the Average Scattering Angle

A knowledge of the average beta particle scattering angle from the gold scattering foil was necessary since Sherman's function is angle dependent. The scattering angle was weighted by the differential scattering cross section, $d\sigma(\theta)/d\Omega$, which was obtained by fitting an exponential to Sherman's tabulated values (60) over the angular range from 105° to 135°. The average value was then determined by means of

$$\langle \theta \rangle = \frac{\int_{\theta_1}^{\theta_2} \theta \frac{d\sigma(\theta)}{d\Omega} d\Omega}{\int_{\theta_1}^{\theta_2} \frac{d\sigma(\theta)}{d\Omega} d\Omega} \quad (2-5)$$

where θ is the scattering angle, $\theta_1 = 110^\circ$, and $\theta_2 = 130^\circ$. The solid angle increment subtended by the beta counter at the scattering foil, $d\Omega$, depended upon θ which necessitated a numerical evaluation of the integrals. The integrals were performed with the aid of the Burroughs 220 digital computer at the Rich Electronic Computer Center. The

average scattering angle found in this fashion was 118.9° . (A less refined division of the differentials gave an average value of 118.8° .)

5. Correction for Instrumental Asymmetries

Stray magnetic fields or causes of undetermined origin could introduce instrumental asymmetries which might give a false effect or cancel a real effect. The experiment was therefore repeated with a thick gold scattering foil - 0.1 mil. Because of plural scattering, as discussed in Chapter I, any polarization effects disappear, and any asymmetries in the beta particle scattering are, therefore, the effect of instrumental imperfections. Table 2 shows the result of this measurement. In order to correct for the instrumental asymmetry the resultant asymmetry is taken as the quotient of the thin foil asymmetry and the thick foil asymmetry. As a further precaution the chamber was rotated 90° about a vertical axis within its supporting frame, and the entire experiment was repeated. The two measured and corrected results appear in Table 2 along with their average value.

6. Calculation of the Polarizations

Knowing Sherman's function, $S(\theta = 118.9^\circ, W = 266 \text{ keV}) = -0.43$, and the normalized, real, beta-gamma coincidence rates at each of the angles, it is possible to calculate the polarizations using Equations (2-2) and (2-3). These results appear in Table 2.

7. Correction of the Polarization for the Finite Angular Resolution of the Detectors

The experimentally determined polarization represented a correlation that was the average of the true correlation over the finite angles subtended by the two detectors. Therefore, $P_{||}$ and P_{\perp} had to

Table 2. Results of Transverse Polarization Experiment

First Chamber Position:			
	Thick Foil:	Thin Foil:	Resultant:
$\frac{I(180^\circ)}{I(0^\circ)}$	0.999 ± 0.009	0.994 ± 0.008	0.995 ± 0.017
$\frac{I(270^\circ)}{I(90^\circ)}$	0.982 ± 0.009	1.009 ± 0.008	1.027 ± 0.017
Second Chamber Position:			
	Thick Foil	Thin Foil:	Resultant:
$\frac{I(180^\circ)}{I(0^\circ)}$	1.009 ± 0.008	1.009 ± 0.009	1.000 ± 0.017
$\frac{I(270^\circ)}{I(90^\circ)}$	0.998 ± 0.008	1.017 ± 0.009	1.019 ± 0.017

$$I(180^\circ)/I(0^\circ) = 0.998 \pm 0.012$$

$$I(270^\circ)/I(90^\circ) = 1.023 \pm 0.012$$

Uncorrected Polarizations:

$$P_{\perp} = -0.002 \pm 0.014$$

$$P = 0.026 \pm 0.014$$

Correction Factors:

$$\eta_{\text{Rose}} = 1.03$$

$$\eta_{\sin \theta \cos \theta} = 1.03$$

$$\eta_{\text{p. s.}} = 1.9$$

Polarizations Corrected for Foil Thickness and Geometry:

$$P_{\perp} = -0.004 \pm 0.028$$

$$P_{\parallel} = 0.052 \pm 0.028$$

be corrected for this finite angular resolution of the detectors. This geometrical correction was made according to an adaptation of the method given by Rose (61). Details of this correction are given in Appendix B. The calculated correction factor is

$$\eta_{\text{Rose}} = 1.03. \quad (2-6)$$

Theoretical expressions for the polarizations contain the factor $\sin \theta \cos \theta$ where θ is the angle between the beta and gamma ray momenta. The measured effect would therefore be attenuated by another factor which expresses the deviation of the average value of $\sin \theta \cos \theta$ from $\sin 135^\circ \cos 135^\circ$. The average value of the trigonometric function over the solid angles subtended by the NaI crystal and the beta particle collimator orifice was computed by

$$\langle \sin \theta \cos \theta \rangle = \frac{\int_{120^\circ}^{150^\circ} \sin \theta \cos \theta \, d\Omega_\beta \, d\Omega_\gamma}{\int_{120^\circ}^{150^\circ} d\Omega_\beta \, d\Omega_\gamma} \quad (2-7)$$

All approximations used in the evaluation of this expression were accurate to second order. The result is $\langle \sin \theta \cos \theta \rangle = 0.487$ which means the polarization correction for this effect is

$$\eta_{\sin \theta \cos \theta} = 1.03. \quad (2-8)$$

8. Depolarization Correction

Theoretical calculations concerning the attenuation of the polarization effect in Mott scattering have been made by H. Wegener (46). Using the conditions of this experiment ($v/c = 0.754$, $d\rho = 0.87 \times 10^{-3} \text{ gm/cm}^2$, $\theta_s = 118.9^\circ$ where v/c is the speed of the beta particle divided by the

speed of light, dp is the foil thickness times the density of gold, and θ_s is the average beta particle scattering angle) one finds that the observed polarization should be corrected by the factor

$$\eta_{p.s.} = 1.9. \quad (2-9)$$

A. R. Brosi et al. (25) have measured the longitudinal polarization of P^{32} beta particles of kinetic energy 616 keV ($v/c = 0.89$). The scattering asymmetry was measured for fifteen different gold target thicknesses ranging from 0.116 (mg/cm^2) to 12.64 (mg/cm^2). The asymmetry for single scattering was then determined by extrapolation of these data to zero foil thickness. The observed attenuation of the polarization of 616 keV electrons in gold targets for $\theta_s = 135^\circ$ is about 30 percent larger than that estimated from the equation of Wegener.

The theoretical figures are suspect since, in a double scattering process, logarithmic divergences arise if either of the two scattering angles goes to zero or if the first scattering is into the plane of the foil. Wegener has more recently (47) applied the Bethe-Fermi Theory (62) of small angle scattering to these regions, thereby eliminating both divergences. These new calculations are not, as yet, in the literature, but Wegener states that they are now in agreement with experiment. In the absence of the new calculation it was necessary to employ Wegener's old results in the interpretation of the present data. Thus the depolarization correction factor $\eta_{p.s.} = 1.9$ was used. This correction is, perhaps, somewhat too low.

9. Probable Error

The errors quoted in the results tabulated in Table 2 are, essentially,

probable errors computed according to the usual procedures (63). The error calculation is based mainly on the number of real coincidences obtained at one counting position, about 24,000 coincidences. This number of coincidences represents the sum of the counts obtained at one counting position (e.g. the 90° position) with the apparatus positioned in its original configuration plus the counts obtained at the same counting position with the apparatus reoriented at 90° with respect to the initial configuration.

10. Correction of the Polarizations

Knowing the three corrections and the uncorrected polarizations, the corrected polarizations were calculated and appear in Table 2. The corrected polarizations are:

$$\begin{aligned} P_{\perp} &= -0.004 \pm 0.028, \\ P_{\parallel} &= 0.052 \pm 0.028. \end{aligned} \tag{2-10}$$

CHAPTER III

INTERPRETATION OF RESULTS

The purpose of this investigation was to determine the transverse polarization of the Tm^{170} beta particles and to consider the interpretation of the result in terms of the matrix element parameters which govern the decay.

The theory of first forbidden beta decay as given by M. Morita and R. S. Morita (30) and T. Kotani (8) is used in this section. The notation in general is that of Kotani. For spin change of one, the nuclear parameters are x , u , and Λ . These parameters are ratios of the various matrix elements and are defined as follows:

$$x = -C_V \int \vec{r} / C_A \int B_{ij} ,$$

$$u = \int i \vec{\sigma} \times \vec{r} / \int B_{ij} ,$$

$$\Lambda = \int i \vec{\sigma} / \xi \int \vec{r} ,$$

where $\xi = \alpha Z / 2\rho$, $\alpha = 1/137$ (the fine structure constant), Z is the atomic number of the daughter nucleus, and ρ is the nuclear radius. C_V and C_A are the vector and axial vector beta decay coupling constants, respectively, and the integrals represent the nuclear matrix elements in the terminology of Konopinski and Uhlenbeck (32). For Yb^{170} $\xi = 14.77$ if one assumes that the nuclear radius is given by $0.43 \alpha A^{1/3}$ where A is the atomic mass number.

H. G. Dulaney (18) has found values of x , u , and Λ which give the observed shape correction factor (Tm^{170} has an allowed shape (13), (14), (15)) and the observed beta-gamma directional correlation coefficient (15), (17) as computed using Kotani's formulas (8). The values of the polarization given by these formulas for representative values of the matrix element parameters at a beta energy of 266 keV are given in Table 3 along with the observed polarization from Chapter II. A careful study of the polarization predicted by Kotani (8) for Dulaney's acceptable parameters (18) indicates that the predicted parallel component of the polarization at 266 keV should be negative and of the order of a few percent, as evidenced in the representative values listed in Table 3. The observed polarization does not disagree in magnitude with this prediction, but the observed sign is probably positive.

The probable error quoted for the measured value of the polarization does not preclude either sign of polarization. For a polarization near zero it will be recognized that it becomes exceedingly difficult to reduce the statistical error to an extent which permits an unambiguous determination of the algebraic sign.

R. W. Newsome, Jr. and H. J. Fischbeck (64) have used the formulas of Morita and Morita (30) and the exact electron radial wave functions calculated by Bhalla and Rose (65) to calculate some first forbidden beta decay observables. Newsome and Fischbeck found the elimination of two customary approximations, i.e., the neglect of the finite nuclear size and the neglect of terms of order $(\alpha Z)^2$ in an expansion of the electron radial wave functions, changed the predicted values of the observables somewhat drastically for certain choices of matrix element parameters.

Table 3. Theoretical and Observed Polarizations

x	u	Λ	$P_{ }$	P_{\perp}
-0.22	-0.04	2.50	-.027	.011
-0.12	-0.04	2.50	-.048	.020
-0.22	0.00	2.50	-.039	.010
-0.12	0.00	2.50	-.022	.016
-0.22	0.04	2.50	-.022	.009
-0.12	0.04	2.50	-.032	.014
-0.28	0.08	2.00	-.025	.011
-0.20	0.08	2.00	-.029	.012
-0.42	0.00	2.00	-.024	.010
-0.26	0.00	2.00	-.033	.014
-0.42	-0.08	2.00	-.029	.012
-0.26	-0.08	2.00	-.046	.019
-0.80	0.18	1.50	-.027	.011
-1.10	0.12	1.50	-.028	.012
-0.50	0.12	1.50	-.032	.013
-1.00	0.04	1.50	-.032	.014
-1.60	0.00	1.50	-.030	.013
-1.80	-0.06	1.50	-.031	.013
-1.20	-0.06	1.50	-.036	.015
-1.90	0.34	1.30	-.033	.014
-1.00	0.30	1.30	-.031	.013
-1.90	0.26	1.30	-.036	.015
-1.00	0.22	1.30	-.036	.015
4.20	0.24	1.30	-.046	.019
5.20	0.30	1.30	-.046	.020
5.20	0.18	1.30	-.043	.018
-0.20	0.00	2.53	-.026	.011
-1.05	0.05	1.58	-.027	.011
-11.50	0.00	1.38	-.033	.013
4.00	0.20	1.26	-.056	.023
13.00	0.00	1.35	-.033	.014
Observed Polarization			.052 \pm .028	-.004 \pm .028

Their work includes the shape correction factor, the beta-gamma directional correlation coefficient, and the beta-circularly polarized gamma correlation for La^{140} and Ga^{72} .

For the range of acceptable matrix element parameters found by Dulaney (18) for the decay of Tm^{170} the shape correction factor, the beta-gamma directional correlation coefficient, and the beta-circularly polarized gamma correlation coefficient were computed using both the Kotani formulas (8) and the Morita and Morita formulas (30) employing the exact radial wave functions (65). (It is noted that the beta-circularly polarized gamma correlation has not yet been measured for the decay of Tm^{170} .) In this instance the approximations used by Kotani do not substantially alter these observables as given by the more exact expressions. It was not feasible to investigate the use of exact electron radial wave functions for the calculation of the transverse polarizations because at present the literature does not contain explicit formulas for any of the electron polarizations that permit immediate use of exact radial wave functions. Such formulas would be valuable tools for beta decay analysis, and it is definitely indicated that such formulas should be obtained.

Finally, there may be ambiguity in the comparison of experiment and theory regarding the sign of the transverse polarizations. Unfortunately, although the difficulty is recognized it is not possible for us to definitely preclude an error in sign in the comparison of the experiment and the theoretical polarization expression. The principal problem is the sparsity of transverse beta polarization experiments; there are so far no cases in which any theoretical expression for the beta transverse polarization can be unambiguously compared with experiment.

Although not directly concerned with the present investigation, the predicted values of the beta-circularly polarized gamma correlation coefficient, which were computed as a part of the investigation on the use of the exact electron radial wave functions, are given in Table 4 for representative values of the matrix element parameters found by Dulaney (18). This coefficient is given in Kotani's approximation (8) and by use of the formulas of Morita and Morita (30) with the wave functions of Bhalla and Rose (65). The coefficients are calculated for an angle between beta and gamma of 160° and for a beta energy of 450 keV. The tabulation may be of help in ascertaining the utility of such a measurement in further delimiting the matrix element parameters. Examination of the table reveals that the beta-circularly polarized gamma correlation coefficient is insensitive to variation of matrix element parameters within the range suggested by Dulaney (18). At present the measurement of the coefficient gives experimental error limits the order of 20 percent.

Table 4. Theoretically Calculated Beta-Circularly Polarized Gamma Correlation Coefficients

x	u	Λ	$\omega(\text{Kotani})$	$\omega(\text{Morita})$
-0.22	-0.04	2.5	.60	.59
-0.12	-0.04	2.5	.67	.65
-0.22	0.00	2.5	.60	.59
-0.12	0.00	2.5	.65	.64
-0.22	0.04	2.5	.59	.58
-0.12	0.04	2.5	.63	.62
-0.28	0.08	2.0	.60	.59
-0.20	0.08	2.0	.62	.61
-0.42	0.00	2.0	.59	.58
-0.26	0.00	2.0	.62	.60
-0.42	-0.08	2.0	.60	.58
-0.26	-0.08	2.0	.65	.62
-0.80	0.18	1.5	.61	.58
-1.10	0.12	1.5	.61	.57
-0.50	0.12	1.5	.63	.60
-1.00	0.04	1.5	.62	.58
-1.60	0.00	1.5	.61	.57
-1.80	-0.06	1.5	.61	.57
-1.20	-0.06	1.5	.62	.58
-1.90	0.34	1.3	.62	.58
-1.00	0.30	1.3	.62	.59
-1.90	0.26	1.3	.63	.58
-1.00	0.22	1.3	.63	.59
4.20	0.24	1.3	.64	.55
5.20	0.30	1.3	.64	.56
5.20	0.18	1.3	.63	.56

CHAPTER IV

RECOMMENDATIONS FOR FURTHER RESEARCH

The apparatus developed for the experiment is suitable for the measurement of transverse polarizations in other isotopes. Any isotope that decays with emission of a beta particle followed by a gamma ray is acceptable provided the energies are high enough for the electronic circuits to function properly. Some suitable first forbidden beta emitters for which other observables (shape, beta-gamma directional correlation, etc.) are known follow: Sb^{124} , Eu^{152} , Eu^{154} , Re^{186} , and Re^{188} .

The apparatus is inherently not symmetric for the measurement of the transverse polarization perpendicular to the plane of the beta and gamma. Furthermore, on theoretical grounds, this component is expected to be reduced by the factor $3\alpha Z/4$ (where Z is the atomic number of the daughter nucleus and $\alpha = 1/137$) compared to the parallel component. Hence, it is recommended that counting time be devoted primarily to the measurement of the parallel component.

In order to improve the symmetry of the experiment and thus better ensure the absence of instrumental asymmetries it is recommended that the following modifications in the apparatus be made:

(a) rotate the gold scattering foil about a vertical axis at some constant angular velocity in order to assure an average uniformity in foil thickness,

(b) install a second beta counter at a 180 degree angle to the present counter; this will also enable data to be taken at twice the present rate.

After making these modifications it is recommended that measurement of the parallel component of the polarization in the Tm^{170} beta decay be repeated in order to secure more evidence for the sign of the polarization.

Formulas for the electron polarizations in first forbidden beta decay should be derived that use the exact electron radial wave functions that have been calculated by Bhalla and Rose (65). A reasonable starting point would be expressions given by H. A. Weidenmüller (67).

A P P E N D I C E S

APPENDIX A

MOTT SCATTERING

The detailed theory of Mott scattering was first worked out by N. F. Mott (40), (41), (37). The results which appear here are in a form which makes them applicable to beam polarization experiments in particular. The scattering angles are defined in Figure 10.

Consider the motion of electrons in a central field governed by the Dirac equation (68), (69). The wave function is a four component spinor which must have the asymptotic form

$$\psi_{\lambda}^{\pm} = a_{\lambda} e^{ikz} + r^{-1} e^{ikr} u_{\lambda}(\theta, \varphi), \quad (\lambda = 1, 2, 3, 4). \quad (A-1)$$

The a_{λ} are amplitudes describing the incoming wave, the u_{λ} are amplitudes describing the outgoing wave, and k is 2π times the electron momentum divided by Planck's constant. The differential scattering cross-section $I(\theta, \varphi)d\omega$ is given by

$$I(\theta, \varphi)d\omega = \left\{ \sum_1^4 |u_{\lambda}(\theta, \varphi)|^2 / \sum_1^4 |a_{\lambda}|^2 \right\} d\omega. \quad (A-2)$$

the free particle solutions of the Dirac equation are (70), (37)

$$\begin{aligned} \psi_1 &= - \frac{Ap_3 + B(p_1 - ip_2)}{mc + W/c} S, \quad \psi_3 = AS, \\ \psi_2 &= - \frac{A(p_1 + ip_2) - Bp_3}{mc + W/c} S, \quad \psi_4 = BS, \\ S &= \exp(i(p_1x + p_2y + p_3z - Wt)/\hbar). \end{aligned} \quad (A-3)$$

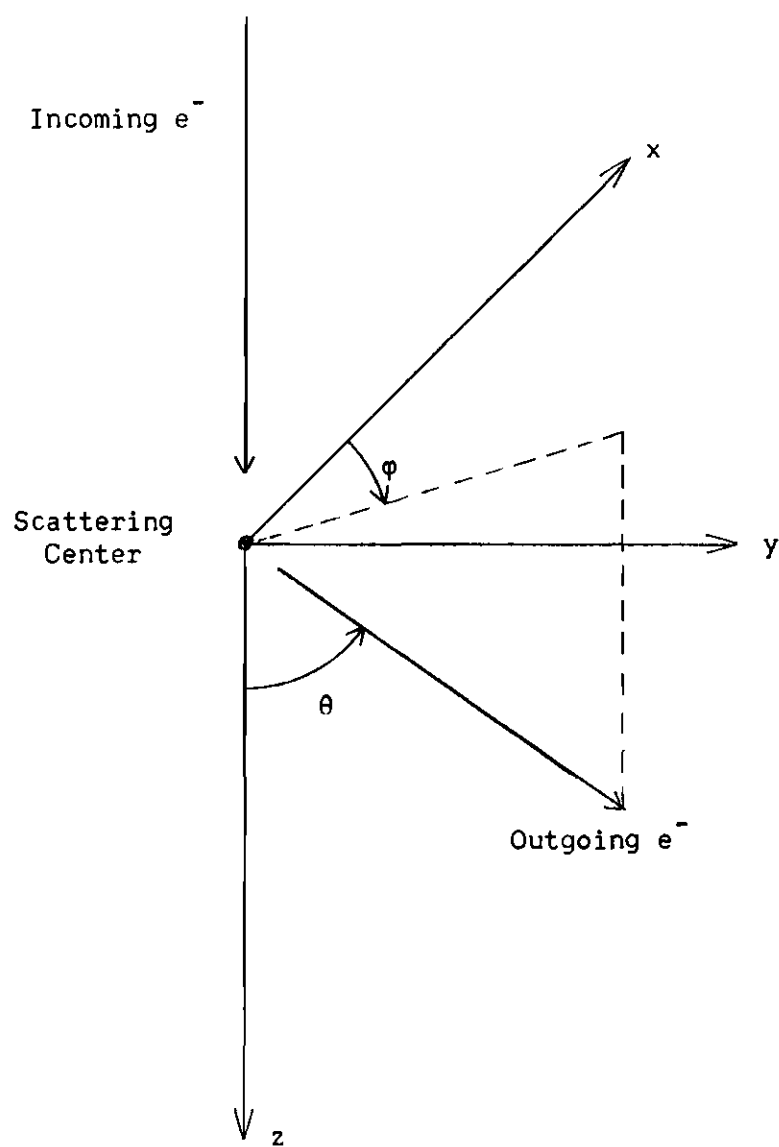


Figure 10. Mott Scattering Event.

Here p_1 , p_2 , and p_3 are the x , y , and z components, respectively, of the beta particle momentum, m is the rest mass of the electron, c is the speed of light, W is the electron kinetic energy, \hbar is Planck's constant divided by 2π , and A and B are arbitrary constants.

Since the asymptotic form of the solution to the Dirac equation is given by these free particle solutions we find:

$$\left| \frac{a_1}{a_3} \right| = \frac{\hbar c k}{W + mc^2} = \left| \frac{a_2}{a_4} \right|. \quad (\text{A-4})$$

Asymptotically the scattered wave may be regarded as made up of a number of plane waves proceeding outward from the scattering center in different directions; hence, the same relation exists between the u_λ 's. The differential scattering cross section now reduces to

$$I(\theta, \varphi) d\omega = [(|u_3|^2 + |u_4|^2) / (|a_3|^2 + |a_4|^2)] d\omega. \quad (\text{A-5})$$

An incoming beam of a particular polarization may be regarded as made up of appropriate numbers of electrons with spins respectively parallel and antiparallel to the direction of propagation. First consider the scattering in these two particular cases. The asymptotic forms of the components ψ_3 and ψ_4 are

$$\left. \begin{aligned} \psi_3 &\sim e^{ikz} + r^{-1} e^{ikr} f_1(\theta, \varphi) \\ \psi_4 &\sim r^{-1} e^{ikr} g_1(\theta, \varphi) \end{aligned} \right\} \quad (\text{A})$$

$$\left. \begin{aligned} \psi_3 &\sim r^{-1} e^{ikr} g_2(\theta, \varphi) \\ \psi_4 &\sim e^{ikz} + r^{-1} e^{ikr} f_2(\theta, \varphi) \end{aligned} \right\} \quad (\text{B})$$
(A-6)

for the two cases, (B) referring to electrons with spins parallel and (A) with spins antiparallel to the direction of incidence. To obtain the functions f_1, g_1, f_2, g_2 use may be made of the sets of solutions found by Darwin (71) of the Dirac equation with the scalar scattering potential V a function of r only and the vector potential zero:

$$\left. \begin{aligned} \psi_3 &= (n+1) G_n P_n(\cos \theta), \quad \psi_4 = -G_n P_n^1(\cos \theta) e^{i\varphi} \\ \psi_3 &= n G_{n-1} P_n(\cos \theta), \quad \psi_4 = G_{-n-1} P_n^1(\cos \theta) e^{i\varphi} \end{aligned} \right\} \quad (A)$$

$$\left. \begin{aligned} \psi_3 &= G_n P_n^1(\cos \theta) e^{i\varphi}, \quad \psi_4 = (n+1) G_n P_n(\cos \theta) \\ \psi_3 &= -G_{-n-1} P_n^1(\cos \theta) e^{i\varphi}, \quad \psi_4 = n G_{-n-1} P_n(\cos \theta) \end{aligned} \right\} \quad (B)$$
(A-7)

Here $P_n^m(\cos \varphi)$ are the associated Legendre polynomials. G_n is a solution of the simultaneous equations

$$\begin{aligned} \frac{1}{\hbar} \left(\frac{W}{c} - \frac{eV}{c} + mc \right) F_n + \frac{dG_n}{dr} - \frac{n}{r} G_n &= 0 \\ -\frac{1}{\hbar} \left(\frac{W}{c} - \frac{eV}{c} - mc \right) G_n + \frac{dF_n}{dr} + \frac{n+2}{r} F_n &= 0 \end{aligned}$$
(A-8)

and G_{-n-1} is a solution of a similar pair of equations with $-n-1$ in place of n . There are two solutions for each case because the Dirac equation is second order. In order to obtain the proper solution of the scattering problem one must take a linear combination of the two independent solutions which satisfies the boundary conditions of the problem.

It follows that the proper solutions G_n, G_{-n-1} of (A-7) can

be taken to have the asymptotic forms

$$\begin{aligned} G_n &\sim r^{-1} \sin \left(kr - \frac{1}{2} n \pi + \eta_n \right) , \\ G_{-n-1} &\sim r^{-1} \sin \left(kr - \frac{1}{2} n \pi + \eta_{-n-1} \right) . \end{aligned} \quad (\text{A-9})$$

where η_n and η_{-n-1} are arbitrary constants, denoted as phases, which are determined by the boundary conditions. One now solves (A-8) for G_n and G_{-n-1} using (A-9) as the boundary condition. Next solutions (A-7) for Ψ_λ are found which have asymptotic forms (A-6). From these solutions and the form of (A-6) one obtains f_1 , f_2 , g_1 , and g_2 such that for the case (B) of parallel spins

$$f_2(\theta, \varphi) = f_1(\theta, \varphi) = f(\theta) , \quad (\text{A-10})$$

and

$$\begin{aligned} g_2(\theta, \varphi) &= -g(\theta)e^{-i\varphi} , \\ g_1(\theta, \varphi) &= g(\theta)e^{i\varphi} . \end{aligned} \quad (\text{A-11})$$

The case of the incident wave given by

$$\psi_3 = Ae^{ikz}, \quad \psi_4 = Be^{ikz} \quad (\text{A-12})$$

may be obtained, by linear combination, to give

$$\begin{aligned} u_3 &= Af - Bge^{i\varphi} , \\ u_4 &= Bf + Age^{i\varphi} , \end{aligned} \quad (\text{A-13})$$

so that

$$I(\theta, \varphi) = |f|^2 + |g|^2 + (fg^* - gf^*) \left[\frac{-AB^* e^{i\varphi} + A^* B e^{i\varphi}}{|A|^2 + |B|^2} \right] \quad (A-14)$$

or

$$I(\theta, \varphi) = \frac{d\sigma(\theta)}{d\Omega} \left[1 + \frac{(fg^* - gf^*)}{|f|^2 + |g|^2} \left\{ \frac{-AB^* e^{i\varphi} + A^* B e^{-i\varphi}}{|A|^2 + |B|^2} \right\} \right] \quad (A-15)$$

The amplitude of the φ dependent part of the scattering intensity is dependent on g which is proportional to $\sin(\eta_k - \eta_{-k-1})$. Setting $\eta_k = \eta_{-k-1}$ is equivalent to neglecting relativity and spin; this makes $g = 0$ which removes the φ dependence. The asymmetry is then a manifestation of the spin (magnetic moment) -orbit (electron current) interaction.

Suppose one wishes to compare the scattering intensity at given θ and φ to that at θ and $\varphi + \pi$. (See Figure 10.)

$$\frac{I(\theta, \varphi)}{I(\theta, \varphi + \pi)} = \frac{1 - S(\theta)P(\varphi)}{1 + S(\theta)P(\varphi)} \quad (A-16)$$

where

$$S(\theta) = i \frac{fg^* - gf^*}{|f|^2 + |g|^2}, \quad P(\varphi) = i \frac{A^* B e^{-i\varphi} - AB^* e^{i\varphi}}{|A|^2 + |B|^2}.$$

Since $|S(\theta)P(\varphi)| \ll 1$ we may write

$$I(\theta, \varphi)/I(\theta, \varphi + \pi) = 1 - 2S(\theta)P(\varphi). \quad (A-17)$$

$S(\theta)$ is called "Sherman's function." For the case of electrons scattered by an unshielded atomic nucleus of charge Ze so that $v = -Ze/r$, Noah Sherman (60) has numerically evaluated $S(\theta)$ and $d\sigma(\theta)/d\Omega$. Shin-R Lin (72) has recently given Sherman's function and

the differential scattering cross section corrected for screening by the atomic electrons. The calculations for the present work were mostly performed prior to the appearance of the latter results. For the conditions of the present experiment the comparison between the two results is as follows:

electron energy = 266 keV

scattering angle = 120°

$d\sigma/d\Omega$		S	
Sherman	Lin	Sherman	Lin
5.73×10^2	6.40×10^2	-0.433	-0.422

It is clear that use of Sherman's results rather than Lin's does not have a significant effect here.

APPENDIX B

ANGULAR RESOLUTION CORRECTION

The results of an angular correlation measurement are conveniently expressed in terms of a Legendre polynomial expansion. That is, the coincidence counting rate per unit solid angle is proportional to

$$W(\theta) = \sum_{v=0}^{v_m} \alpha_v P_v(\cos \theta) \quad (\text{B-1})$$

where θ is the angle between the momenta of the two radiations that are detected in the experiment. When the detectors for the two radiations subtend finite solid angles Ω_1 and Ω_2 at the source, it is necessary to correct the observed correlation, which is somewhat attenuated, prior to comparison with the theoretical correlation. The geometry envisaged is shown in Figure 11. The detectors (scintillation counters) are assumed to be crystals cut in the form of right circular cylinders with the bases oriented towards the source. The source, at the origin, is on the intersection of the axes of the cylinders. We will assume that the absorption coefficients are infinite. For the gamma energy in the present experiment, 84 keV, this is a good approximation and it is, of course, a proper assumption for the beta detector. M. E. Rose (61) demonstrates that the form of the correlation function is unchanged, and each coefficient α_v must be multiplied by an attenuation factor Q_v ($v > 0$). Rose shows that if γ is the half-angle subtended by the front face of the detector at the source,

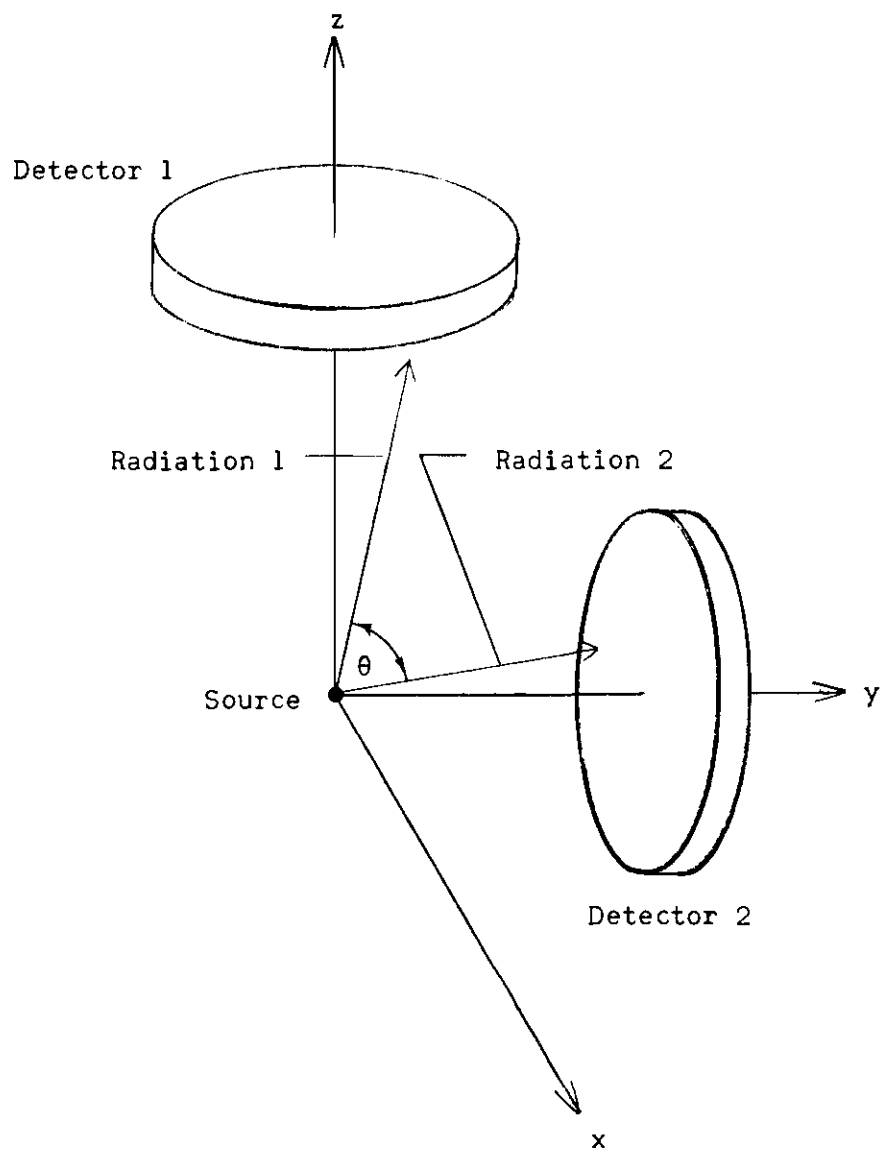


Figure 11. Finite Angular Resolution.

$$Q_v = \frac{P_{v-1}(\cos \gamma_1) - \cos \gamma_1 P_v(\cos \gamma_1)}{(v+1)(1 - \cos \gamma_1)} \cdot \frac{P_{v-1}(\cos \gamma_2) - \cos \gamma_2 P_v(\cos \gamma_2)}{(v+1)(1 - \cos \gamma_2)} \quad (\text{B-2})$$

where the subscripts refer to the two radiations and their respective detectors.

This work is adapted to polarization experiments with the aid of Figure 12. According to equation (A-15) in Appendix A the intensity of the scattered beta particle beam may be expressed as

$$I(\phi') = (\text{Constant})[1 + (\text{Constant})P \cos \phi'] \quad (\text{B-3})$$

where P is the beta particle polarization and ϕ' is measured azimuthally about the chamber axis and measures the orientation of the plane containing the scattered beta momentum and the z axis relative to a reference plane perpendicular to the xy -plane.

The correction for finite angular resolution is not large and needs to be carried to only first order which permits simplifying assumptions. The problem may be visualized by placing an observer above the scattering chamber, i.e. placing an observer on the $-z$ axis who looks in the $+z$ direction. Because of the finite size of the two radiation detectors a finite range for the azimuthal angle ϕ' is covered, and this is the effect to be corrected for. The procedure for adapting the results given by Rose (61) to the present geometry then suggests itself: (a) since second order effects in evaluation of the correction are to be ignored one assumes that all beta particles incident on the scattering foil travel parallel to the chamber axis, the $+z$ axis, (b) the effective distances from the source to the detectors are the perpendicular

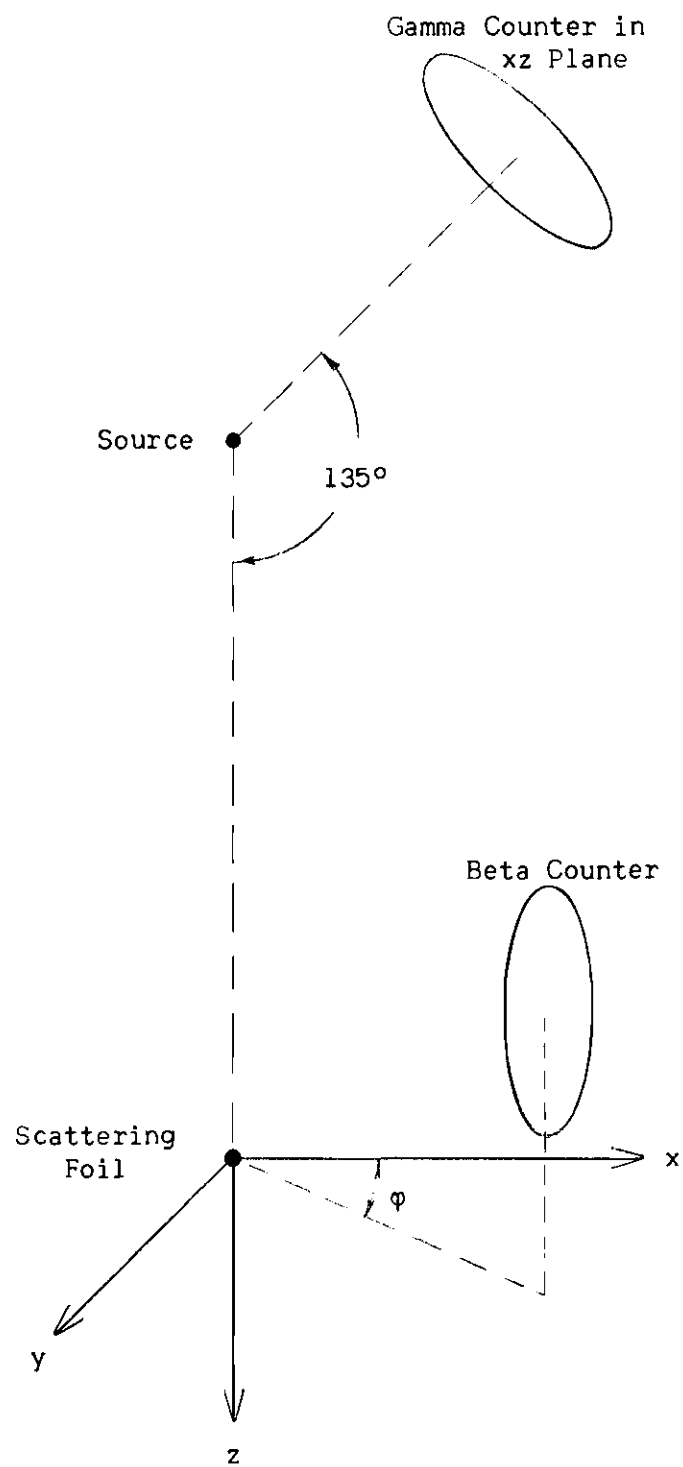


Figure 12. Polarization Experiment Geometry.

distances from the detectors to the z axis, and (c) the fact that the axis of the gamma detector is not perpendicular to the z axis may be disregarded.

Equation (B-3) is already in the form of (B-1) which implies that only the correction factor Q_1 is necessary. Since Q_1 is an attenuation factor of the coefficient of $\cos \theta$ in (B-3) the measured polarization will need to be corrected by $\eta_{\text{Rose}} = 1/Q_1$. For the detector dimensions used in the present experiment, shown in Figure 6 ($\gamma_\beta = 10.8^\circ$ and $\gamma_\gamma = 15.2^\circ$), equation (B-2) may be evaluated to give $Q_1 = 0.97$ so that

$$\eta_{\text{Rose}} = 1.03 . \quad (\text{B-4})$$

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VITA

Robert Edgar Wood was born May 16, 1938, in Philadelphia, Pennsylvania. He attended the public schools of Marietta and Atlanta, Georgia, and graduated in 1956 from Henry Grady High School in Atlanta. In 1960 he received the B. S. degree in Physics from the Georgia Institute of Technology; in 1962 he received the M. S. degree in Physics from the same institution. Since 1962 he has had a half-time teaching assistantship in the School of Physics at the Georgia Institute of Technology. During 1962-1964 he was the recipient of a Graduate Division fellowship. He married Lorna Jo Jarrell in 1962, and they have one child, Laura Ellen Wood. He is a member of Sigma Pi Sigma and Sigma Xi.